U.D.K. 536.423

SOLUTION OF THE ENERGY EQUATION OF A TWO-PHASE MEDIUM TAKING INTO ACCOUNT HEAT TRANSFER BETWEEN PHASES

Djalilova Turgunoy Abdujalilovna
candidate phys.-math. Sci., Associate Professor of Department of Higher Mathematics,
Andijan machine-building institute
tdjalilovaturg'unoy@gmail.com

Atabayev Kamil
candidate phys.-math. Sci., Associate Professor of Department of General Technical Sciences,
Andijan machine-building institute.
K.ATABAYEV@gmail.com

Komolova Gulkhayo Shukirillayevna
assistant of the Department of Higher Mathematics,
Andijan machine-building institute
komolovagulhayo@gmail.com

Annotatsiya. Ushbu maqolada tovushdan tez harakat qiluvchi gaz va qattiq zarrachlarning oqish masalasi ko'rib chiqiladi. Izlanish natijasida zarrachali gazning soplalarda oqishi davomida yengil fazaning devor osti sohasi aniqlangan. Sonli hisoblashlar aniq misollar orqali bajarilgan. Olingan natijalar asosida egri burchakli sirt ko`rinishi chizilgan. Bosim tasdiqlanishi va haroratning sirt bo`ylab tarqalishi diametr va zarrachalarning konsentratsiyasi turli qiymatlarda hisoblangan.

Kalit so`zlar. Ikki fazali muhit, supersonik oqim, kam to'lqin, dinamik siljish, issiqlik uzatish, barotrop muhit.

Аннотация. В данной работе рассматривается задача об обтекании газа с твёрдыми частицами со сверхзвуковой скоростью. При исследовании течения газа с частицами в соплях обнаружена пристеночная область легкой фазы. В конкретном примере сделаны численные расчеты и на основе полученных результатов построены форма поверхности криволинейного угла, распределение давления и температуры потока вдоль поверхности при различных значениях диаметра и концентрации частиц.

Ключевые слова: двухфазная среда, сверхзвуковое обтекание, волна разрежения, динамическое скольжение, теплообмен, баротропная среда.

Abstract. In this paper, we consider the problem of supersonic flow around a gas with solid particles. When studying the flow of gas with particles in the snot, a near-wall region of the light phase was found. In a specific example, numerical calculations are made and, on the basis of the results obtained, the shape of the surface of a curvilinear angle, the distribution of pressure and temperature of the flow along the surface at various values of the diameter and concentration of particles are constructed.

Key words: two-phase medium, supersonic flow, rarefaction wave, dynamic sliding, heat transfer, barotropic medium.

Introduction. In this paper, the problem is solved using the energy equation of both single-phase and two-phase media, taking into account heat transfer between phases. Using an interpenetrating model of a multi-velocity continuous medium[1] and equation [2], the problem is solved about the flow around a “curvilinear angle” of more than 180 by a gas flow with solid particles at a supersonic speed (Fig. 1). In a barotropic medium [3], in the case of a rarefaction flow over the surface of the body, two regions are obtained: I- between the characteristic and the dividing line (dotted line) and II- between the dividing line and the solid surface (solid curve). In the study of the flow of gas with particles in the nozzles, the near-wall region of the light phase was found [4-8]. Without taking into account the volume occupied by the particles, the supersonic two-phase flow around a thin airfoil was considered [9] and, in particular, the structure of the rarefaction wave and the near-wall region during dynamic phase slip were investigated.°

The article [13] analyzes the transfer of matter in inhomogeneous porous media taking into account the inhomogeneous distribution of the velocity field.

Research Methodology. In [14], the problem under consideration is of great importance for aviation and rocket and space technology. The article presents a comparative testing of the Chen and Secundov models and the turbulence model based on the dynamics of two fluids for an axisymmetric subsonic jet. $k-\varepsilon\gamma_t-92$

In contrast to [3,9], the above problem is solved using the energy equations of both single-phase and two-phase media, taking into account heat transfer between phases; the kinematic parameters of the gas in region II are determined from the solution of the corresponding boundary value problem, and the temperature is determined from the equation of the gas energy in finite differences. In a specific example, numerical calculations are made and, on the basis of the results obtained, the shape of the surface of a curvilinear angle, the distribution of pressure and temperature of the flow along the surface at various values of the diameter and concentration of particles are constructed.

Analysis and results. Consider a plane supersonic flow around a concave corner of a two-phase medium with an initial velocity U_0... In this case, a rarefaction wave arises, which in the linear formulation degenerates into characteristics, and for a plane stationary flow of a mixture of gas and particles in the absence of external and heat flows, we have the equations of motion, continuity and energy [2]:x-ωy=0

$$\left.\begin{aligned} u_n\frac{\partial u_n}{dx}+v_n\frac{\partial u_n}{dy}&=-\frac{1}{p_{ni}}\frac{\partial p}{\partial x}+\frac{K}{p_n}\sum_{j=1}^{2}(u_j-u_n)\\ u_n\frac{\partial v_n}{dx}+v_n\frac{\partial v_n}{dy}&=-\frac{1}{p_{ni}}\frac{\partial p}{\partial y}+\frac{K}{p_n}\sum_{j=1}^{2}(v_j-v_n)\end{aligned}\right\}. \quad (1)$$

$$\frac{\partial}{\partial x}(p_nu_n)+\frac{\partial}{\partial y}(p_nu_n)=0 \quad (2)$$

$$\left.\begin{aligned} \overrightarrow{V_1}\nabla i_1-\frac{1}{p_{1i}}\overrightarrow{V_1}\nabla p+N=0,\overrightarrow{V_2}\nabla i_2-q=0\\ N=\frac{p_2}{p_1}\left[q+\frac{K}{p_2}(V_2-V_1)^2\right],\ V_n^2=u_n^2+v_n^2, n=1,2\end{aligned}\right\}. \quad (3)$$

Taking into account that a mixture of gas and solid incompressible particles is considered here, we supplement the system with equations of state of the phases (1)-(3)[10]

$$p = R_1 p_{1i} T_1 \ , p_{2i} = const, i_1 = c_1 T_1, i_2 = c_2 T_2 \ , \quad (4)$$

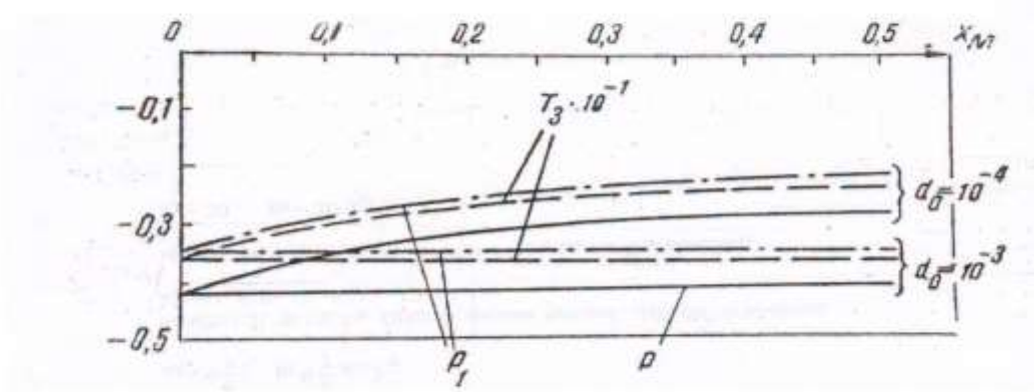
the expression for the function of interphase heat transfer q

$$q = \gamma(T_1 - T_2) \quad (5)$$

and the ratio

$$\frac{p_1}{p_{1i}} + \frac{p_2}{p_{2i}} = 1; \quad (6)$$

Here $p -$ pressure, $u_n, v_n -$ velocities, T_n –temperature, p_{ni}, p_n –true and reduced phase densities, $n -$ й phase, к, $\gamma -$ coefficients of interaction and heat transfer between phases, which in this case are taken as constant, depending on the diameter d_0 and density p_{00} of particles, $R_1 -$ gas constant, $c_1 c_2 -$ heat capacity coefficients.



Pic. 1. The contour of the flow around the corner is larger than the flow of gas with solid particles. 180°

K system valid in region I, the linearization method is applied at (1)-(6)

$$\left. \begin{array}{c} u_n = u_0 + u_n^{`} \ , \ \rho_n = \rho_{n0} + \delta_n \ , \rho_{1i} = \rho_0 + \varepsilon_1 \\ T_n = T_0 + T_n^{`} \ , p = p_0 + p^{`} \end{array} \right\} , \quad (7)$$

where $u_0, \rho_{n0}, p_0, \rho_0, T_0$ –constants; $u_n^{`}, \varepsilon_n, \delta_n, T_n^{`}, p^{`}$ – small values, indices 1 and 2 correspond to the parameters of the gas and particles.

In the case of an irrotational potential flow (1)-(6) taking into account (7) take the form:

$$A_1 \frac{\partial^3 \varphi_1}{\partial x^3} + A_2 \frac{\partial^3 \varphi_1}{\partial x \partial y^2} + A_3 \left(\frac{\partial^3 \varphi_2}{\partial x^3} + \frac{\partial^3 \varphi_2}{\partial x \partial y^2} \right) - A_4 \frac{\partial^2 \varphi_1}{\partial x^2} + A_5 \frac{\partial^2 \varphi_1}{\partial y^2} + A_6 \frac{\partial^2 \varphi_2}{\partial x^2} + A_7 \frac{\partial^2 \varphi_2}{\partial y^2} + A_8 \left(\frac{\partial \varphi_2}{\partial x} - \frac{d \varphi_1}{dx} \right) = 0 \ , \quad (8)$$

$$B_1 \frac{\partial \varphi_1}{\partial x} - B_2 \frac{\partial \varphi_2}{\partial x} = -B_3 (\varphi_1 - \varphi_2) \ ; \quad (9)$$

$\varphi_1, \varphi_2 -$ velocity potentials, $A_i (i = \overline{1,8}), B_j (j = \overline{1,3}) -$ known constant coefficients depending on the Maxa number in the gas, concentration and phase interaction coefficient.

Since the parietal region II is occupied by a gaseous medium, then for the potential of the velocity of the disturbed flow φ_3

$$\varphi_{3yy} = \mu^2 \varphi_{3xx} (\mu^2 = M_1^2 - 1). \quad (10)$$

The pressure and temperature of the flow on a solid surface are found by the Bernoulli and energy equations [11] in finite difference. This approximation of the energy equation will be the more accurate, the smaller the thickness of the near-wall region II.

Let the phase separation line is set straight and forms c and an angle β_0 with the axis x. Obviously, this line is represented as the boundary streamline of particles, through which the gas freely passes into region II. Therefore, for $(8)-(10)$ we have the boundary conditions

$$y=0,\ \varphi_{2y}=-u_0\beta_0,\ \varphi_{1y}=\varphi_{3y},\ \varphi_{1x}=\varphi_{3x}. \quad (11)$$

We add that the velocities of the two-phase system in infinite features are also limited on the characteristic

$$\varphi_1=\varphi_2=0 \qquad (12)$$

On the solid boundary, the condition is satisfied gas environment, i.e. when

$$y=f(x),\varphi_{3y}=-u_0\beta(x),\left[\beta(x)=\frac{df(x)}{dx}\right]; \qquad (13)$$

Here $\beta(x)-$ the angle of inclination of the tangents to the elements of the curvilinear side of the angle, which depends on the shape of the dividing line, the structure of the flow, is an unknown function and must be determined in the process of solving the problem.

Applying to (8), (9) the Laplace transform[12], it is easy to obtain solutions with X (8), (9) satisfying boundary conditions (11) and (12):

$$\varphi_1(x,y)=u_0\beta_0e^{-a_0y}\frac{\rho_{00}}{\rho_0}\sum_{v=0}^{\infty}b_v\left\{\frac{t^{*v+1}}{(v+1)!}+\sum_{x=1}^{\infty}c_x^0\frac{t^{*v+x+1}}{(v+x+1)!}-\frac{k}{\rho_{10}}\frac{\rho_0}{\rho_{20}}\left(\frac{\rho_{00}}{\rho_0}-1\right)\left[\int_0^{t^*}f_1(t^*-\tau)f_3(\tau)d\tau+\sum_{x=1}^{\infty}c_x^0\int_0^{t^*}f_2(t^*-\tau)f_3(\tau)d\tau\right]\right\},$$

$$\varphi_2(x,y)=u_0\beta_0e^{-a_0y}\sum_{v=0}^{\infty}b_v\left[\frac{t^{*v+1}}{(v+1)!}+\sum_{x=1}^{\infty}c_x^0\frac{t^{*v+x+1}}{(v+x+1)!}\right], \qquad (14)$$

where

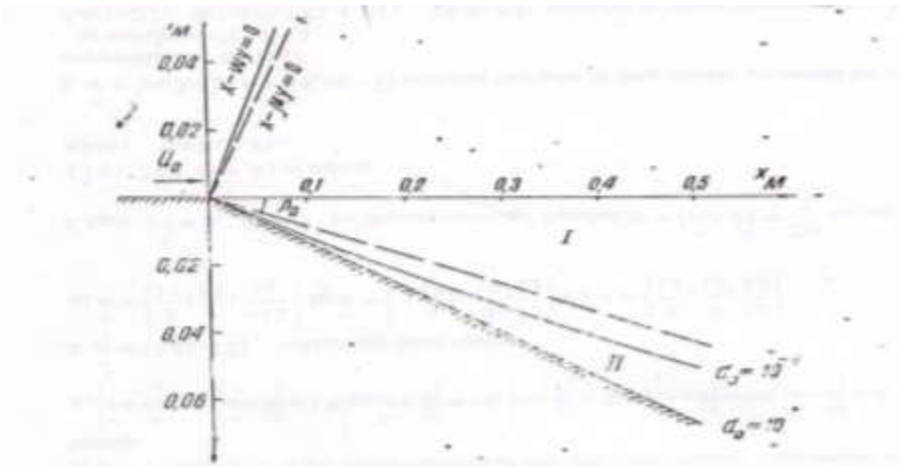
$$t^*=x-wy,w^2=-\frac{A_1B_2+A_2B_1}{A_2B_2+A_3B_1},$$

$$f_1(t^*)=\frac{t^{*v+1}}{(v+1)!},$$

$$f_2(t^*)=\frac{t^{*v+x+1}}{(v+x+1)!}$$

$$f_3(t^*)=e^{-\frac{B_3}{B_1}t^*},$$

$a_0,\alpha_1,\alpha_2,\beta_1,\beta_2,b_v,c_x^0-$ known constant coefficients. Now, taking into account (14) and the equations of motion and energy (1), (3), it is easy to obtain formulas for the pressure and temperature at the phase separation line.



Pic. 2. Distribution of gas flow pressure and gas temperature (dash and dotted line) in the flow region. Equation (10) has a solution:

$$\varphi_3(x,y) = f_1(x-\mu y) + f_2(x+\mu y) \quad (15)$$

functions $f_1(x)$ and $f_2(x)$ taking into account (11), are known from solution (14) in the flow region of a two-phase medium, are not given. Substituting (15) into (13), we obtain a first-order differential equation with respect to, which determines the shape of the solid surface.

The direct problem was solved in a similar way, i.e. at a given value of the angle β_{00} of a solid surface with an axis X , in the process of solving the parameters of regions I, II and the shape of the interface of the phase line are found.

For a specific calculation, consider the case $v_0 = 0$, $x = 1$ and use the Stokes resistance law with cd=24/Re to find the phase interaction coefficient. Then the results for a steam-water mixture [10] at $p_0 = 10$ атм, corresponding to the initial parameters.

$$T_0 = 481 \text{ град}, c_1 = 4{,}8 \cdot 10^3 \text{ м}^2/\text{сек}^2 \cdot \text{град},$$
$$c_2 = 4{,}4 \cdot 10^3 \text{ м}^2/\text{сек}^2 \cdot \text{град}, \beta_0 = 0{,}0875,$$
$$M_1 = 1{,}85 , \rho_{00}/\rho_0 = 1{,}8 , \rho_0 = 0{,}5 \text{ кг} \cdot \text{сек}^2/\text{м}^4,$$
$$\rho_{00} = 0{,}9 \text{ кг} \cdot \text{сек}^2/\text{м}^4, \rho_{10} = 0{,}45 \text{ кг} \cdot \text{сек}^2/\text{м}^4$$

and coefficients K, γ for different values of the particle diameter are shown in Pic. 2. According to calculations, the thickness of the region II depends on the concentration and diameter of the particles, i.e. the finer the particle, the thinner region II, and when it $d_0 = 10^{-5}$см almost disappears, then, apparently, the flow should be considered as one-speed. The parameter of the two-phase flow is less than the parameter of pure gas; therefore, the disturbed region I becomes wider than the disturbed region of pure gas.

Conclusion . Mixture pressure increment curves gas and particles in Pic. 2 in absolute value is higher than the corresponding single-phase flow curves, and the gas temperature distribution curve on a solid surface $(T_3 = T_3^{`}/T_0)$ at $d_0 = 10^{-4}$см is concave relative to the axis and located above the corresponding straight line for $d_0 = 10^{-3}$см.

In conclusion, as a result of the research, the sub-surface area of the light phase during the flow of particulate gas in the nozzles was determined. The results obtained are also plotted and explained graphically. These results can be widely applied to the national economy and the gas industry.

References:

[1]. Rakhmatulin Kh.A. PMM, v. 20, no. 2, 1956, p.184.
[2]. Kraiko A.N., Sternin L.E. PMM, v. 29, no. 3, 1965, p. 418.
[3]. Rakhmatulin Kh.A., Mamadaliev N. PMTF, 1969 No. 4. p. 32.
[4]. Kligel I., Sb. “Questions of rocketry”, 1965, No. 10, p. 3.
[5]. Kligel I., R. Nickerson G. R. Sb. “Detonation and two-phase flow”, M., “Mir”, 1966, p. 183.
[6]. Hoffman Y., D. Thompson X. D. Sb. "Questions of rocket technology", 1967, No. 3, p.46.
[7]. Vershaka L. P. and others “Academy of Sciences of the USSR ”, 1968, No. 3, p. 133.
[8]. Kraiko A.N., Osipov A.A. V, 1968, p. 596.
[9]. Tkalenko R.A. “Academy of Sciences of the USSR”, 1971, No.1, p. 109.
[10]. R.I. Nigmatulin, “Izv. AN SSSR ”, 1967, No. 5, p. 33.
[11]. Kochin N.E., Kibel I.A., Rose N.V. Theoretical Hydromechanics Part 2, Moscow, Fizmatgiz, 1963.
[12]. Ditkin V.A., Kuznetsov P.I. Operational Calculus Handbook, M.-L., Gostekhteoretizdat, 1951.
[13]. Khujayorov B. Kh., Makhmudov Zh. I., Sulaimonov FU, “Filtration and transfer of matter in a cylindrical two-phase porous medium taking into account the inhomogeneity of the filtration velocity field”. Uz. J. PM. No. 4, 2018, art. p. 43-48.
[14]. Malikov Z.M., Navruzov D.P., “Comparison of turbulent models for calculating an axisymmetric submerged jet”.Uzbek. J. PM. No.1, 2021, art. p.p.58-65.
[15]. Loysyanskiy L.G. “Mechanics of liquid and gas”. M. Science, 1987.