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$m-$CONVEX FUNCTIONS AND THEIR PROPERTIES

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Annotatsiya. Ushbu ishda $m-$ qavariq funksiyalarning ba’zi xossalari, jumladan, $m-$qavariq funksiyalarning silliq approksimatsiyasi, $m-$qavariq funksiyalarning ketma-ketligi limitining $m-$qavariqligi, chekli sondagi $m-$qavariq funksiyalar maksimumi $m-$qavariq bo’lishi isbotlanadi.

Kalit so‘zlar: k-tartibli Gessian operatori, sunbgarmonik funksiya, $m-$qavariq funksiya, silliq approksimatsiya, integral o’rta qiymat.

Аннотация. В данной работе изучена $m-$выпуклые функции и их некоторое свойства, в частности аппроксимация $m-$выпуклых функций и доказана $m-$выпуклость лимит последовательностей $m-$выпуклых функций**,** $m-$выпуклость максимума конечного числа $m-$выпуклых функций.

Ключевые слова: оператор Гессиана $k-$порядка, субгармонические функции, $m-$выпуклые функции, гладкая аппроксимация, среднее интегрально значение.

Abstract. In this work studies $m-$convex functions and some of their properties, in particular, approximation of $m-$convex functions and proved the $m-$ convexity of the limit sequences of $m-$convex functions, $m-$convexity of maximum of the finite number of convex functions.

Keywords: k- Hessian operator, subharmonic functions, $m-$convex functions, smooth approximation, mean integral value.

Introduction. Lets imagine we are given $D\subset \mathbb{R}^n$ function in one sphere and $u\in C^2(D)$ function in another. Lets have a look at the matrix $D^2u=\left(\frac{\partial^2 u}{\partial x_i,\partial x_j}\right)$ that was composed of the second-order derivatives of the function. It is measured as a symmatric matrix D^2u because of $u\in C^2(D)$, so all its numerals are considered to be factual numbers. Look at this figures

$$H_k(u)=H_k(\lambda)=\sum_{1\le i_1<\ldots\ldots<i_k\le 1}\lambda_{i_1}\cdot\ldots\cdot\lambda_{i_k}$$

, in this example $\lambda=(\lambda_1,\ldots,\lambda_n)$ is a vektor and D^2u is also a vector that composed of eigenvalues of the matrix. This systematic graph k is called Hessian operator. If we are given such a symbol such as

$$(t+\lambda_1)\ldots(t+\lambda_n)=t^n+H_1(\lambda)t^{n-1}+\ldots+H_n(\lambda),\quad t\in \mathrm{R},$$

in this case we will learn functions that leads to the solution of the task

$$H_k(u(x))=H_k(\lambda(x))\ge 0,\quad \forall x\in D.$$

1st explanation (q. [5]). If the relation between $H_k(u(x)) = H_k(\lambda(x)) \geq 0,\ \forall x \in D,\ k = 1,\dots,n-m+1$ function $u \in C^2(D)$ and between the field $D \subset \mathrm{R}^n$ is acceptable this function is called $m-$ convex function in the field of D.

Convex function class $m-$ is determined by $m-cx(D)$

in another words with the help of the graph given below

$$m-cx \cap C^2(D) = \{u \in C^2(D) : H_k(u(x)) = H_k(\lambda(x)) \geq 0, \quad \forall x \in D, \quad k = 1,\dots,n-m+1\}.$$

Generally, defining convex functions $m-$ is an important issue of modern life. A number of properties of convex quadratic polynomials are described by Trudinger in the class of continuous functions by currents and in the class of $m-$ semicontinuous functions in general as a descending function.We give general explanation to the convex $m-$ just for local integration functions and consider its important properties.

2nd explanation. $u \in L^1_{loc}(D)$ should be a partially uninterrupted function that goes down from the height. If doublé deferentialized $m-$was convex and $v_1,\dots,v_{n-m}$ for functions defined as follows and if the flow was positive

$$\begin{gathered}\left[dd^c u \wedge dd^c v_1 \wedge \ldots \wedge dd^c v_{n-m} \wedge \beta^{m-1}\right](\omega) = \\ = \int u dd^c v_1 \wedge \ldots \wedge dd^c v_{n-m} \wedge \beta^{m-1} \wedge dd^c \omega, \qquad \omega \in F^{0,0}(D \times \mathrm{R}^n)\end{gathered} \tag{1}$$

in this case $u \in L^1_{loc}(D)$ function $D \subset \mathrm{R}^n$ is called convex in $m-$.

The following properties are directly derived from the definition:

1st property. If the function u in D is convex $m-$, in this case $u_j(x) \in C^\infty(D)$ and in the field D convex functions order will be found $m-$, and they are to be $j \to \infty$ and $u_j \downarrow u$.

Prove. Let's look at this standard core function

$$k(x) = \begin{cases} \frac{1}{c} e^{-\frac{1}{1-|x|^2}} & |x| < 1 \\ 0 & |x| \geq 1 \end{cases}$$

Here the constant

$$\int_{R^n} k(x) dV = 1$$

is chosen to satisfy the equality. We construct this sequence using this given below

kernel

$$K_j(x) = \frac{1}{j^m} k(j \cdot |x|), \ j = 1,2,3,...$$

It is known that the sequence of these functions $|x| < \frac{1}{j}$ outside the sphere is equal to zero, respectively. Let's take a look at this package

$$u_j(x)=u*K_j(y-x)=\int_{R^n}u(y)K_j(y-x)dy=\int_{R^n}u(y+x)K_j(y)dy$$

We show that u_j functions in D spheres $m-$ will be convex moreover $j\to\infty$ in the sphere of $u_j\downarrow u$ is also going to be convex.

Indeed, for all $v_1,v_2,\dots,v_m$ $\in C^2(D)\cap m$ - $cv(D)$ functions

$$\left[dd^cu_j\wedge dd^cv_1\wedge\dots\wedge dd^cv_{n-m}\wedge\beta^{m-1}\right]\omega=\int u_j\wedge dd^cv_1\wedge\dots\wedge dd^cv_{n-m}\wedge\beta^{m-1}\wedge dd^c\omega=$$

$$=\int\left(\int_{\mathbb{R}^n}u(y+x)K_j(y)dy\right)\wedge dd^cv_1\wedge\dots\wedge dd^cv_{n-m}\wedge\beta^{m-1}\wedge dd^c\omega=$$

$$=\int_{\mathbb{R}^n}\left(\int u(y+x)\wedge dd^cv_1\wedge\dots\wedge dd^cv_{n-m}\wedge\beta^{m-1}\wedge dd^c\omega\right)K_j(y)dy\geq 0$$

So u_j functions D in the spheres such as $m-$ is considered to be convex. $j\to\infty$ in $K_j(x)\to\delta(x)$ because of Dirak's approach to δ function

$$\lim_{j\to\infty}u_j(x)=\lim_{j\to\infty}u*K_j(y-x)=\lim_{j\to\infty}\int_{R^n}u(y)K_j(y-x)dy=\lim_{j\to\infty}\int_{R^n}u(y+x)K_j(y)dy=$$

$$=\int_{R^n}u(y+x)\delta(y)dy=u(x)$$

And now it is enough to show that it is u_j monotonic decrease $u_j(x)\geq u_{j+1}(x)$ and we use subharmonicity in defining this. Because function $u(x)$ in the sphere of D is considered as a subharmonic function $u_j(x)$ and D. This is based on the cumulative mean

$$u_j(x)\geq u_{j+1}(x)$$

Because

$$u_j(x)=\sigma_m\int_0^{\frac{1}{j}}K_j(t)t^{n-1}M\ (u,x,t)dt=\sigma_n\int_0^{\frac{1}{j+1}}K_{j+1}(\tau)\tau^{n-1}M\ \left(u,x,\frac{(j+1)\tau}{j}\right)d\tau\geq$$

$$\geq\sigma_n\int_0^{\frac{1}{j+1}}K_{j+1}(\tau)\tau^{n-1}M\ (u,x,\tau)d\tau=u_{j+1}(x)$$

relation is reasonable.

2nd property. $m-cx$ decreasing sequence or flat approaching sequence limit namely is $m-cx$.

As a prove of this property, we use the above definition for the direct passage to the limit under the integral.

The main conclusion of the article consists of the following theorem:

Theorem. A finite number of convex functions maximum $m-$ will be convex $m-$function again.

The function $m-$ which was supremum $u(x)=\sup_\theta u_\theta(x)$ which was regulated from above and which was optional flat bounded $\{u_\theta(x)\}$ $m-$can also be called convex function.

For further clarification, for a local plane-bounded sequence $\{u_j\} \subset m-cx$, the upper limit regulation $u(x) = \overline{\lim_{j\to\infty}} u_j(x)$ is $u^*(x)$ also $m-$ a convex function.

Prove. $v_1,\ldots,v_{n-m} \in m-cx(D)\cap C^2(D)$, we assign functions and get $\alpha = dd^c v_1 \wedge \ldots \wedge dd^c v_{n-m} \wedge \beta^{m-1}$. α the differential form is closed and strongly positive. Therefore, $dd^c u \wedge \alpha$ differential operator will be an elliptical operator and $dd^c u \wedge \alpha = 0$ is determined by the equation $\alpha-$ for harmonic functions $B \subset\subset D-$ for the sphere there is Poisson kernel $P_\alpha(z,w)$ which was determined by Grin function $G_\alpha(z,w)$

$$u(z) = \int_{\partial B} P_\alpha(z,w)u(w)d\sigma(w), \;\; z \in B. \tag{2}$$

$$\left[dd^c u \wedge \alpha\right](\omega) = \int u\alpha \wedge dd^c\omega \geq 0, \quad \forall \omega \in F^{0,0}, \;\; \omega \geq 0,$$

and the class of subgarmonic functions defined by the inequality α is determined by requiring this integral inequality to be satisfied instead of equation (2). (q. [3]).

$$u(z) \leq \int_{\partial B} P_\alpha(z,w)u(w)d\sigma(w), \quad z \in B \tag{3}$$

If $\alpha = \beta^{n-1}$ is right, then $\alpha-$ the class of subharmonic functions overlaps with the well-known class of subharmonic functions. As proved in the theory of subharmonic functions, we can show that the relation (3) holds for a maximum of a finite number $u_1(z),\ldots,u_N(z)$ $\alpha-$ of subharmonic functions:

$$\max\left(u_1(z),\ldots,u_N(z)\right) \leq \int_{\partial B} P_\alpha(z,w)\max\left(u_1(w),\ldots,u_N(w)\right)d\sigma(w), \;\; \forall z \in B \;.$$

Hence, $\max\left(u_1(z),\ldots,u_N(z)\right)$ it has a subharmonic function $\alpha-$ in the field D

$$\left[dd^c u \wedge dd^c v_1 \wedge \ldots \wedge dd^c v_{m-1} \wedge \beta^{n-m}\right](\omega) =$$

$$= \int u dd^c v_1 \wedge \ldots \wedge dd^c v_{m-1} \wedge \beta^{n-m} \wedge dd^c \omega \geq 0, \; \forall \omega \in F^{0,0}, \; \omega \geq 0.$$

Hence it follows that this inequality holds at the maximum for a finite number $m-$ of convex functions.

The first part of the theorem is proved. The proof of the second part of the theorem is proved in the class of subharmonic functions, as in potential theory.

Referencies

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