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A MAP OF HOMOGENEOUS FIXED POINTS OF DISCRETE DYNAMIC SYSTEMS

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Annotatsiya. Ushbu maqolada diskret dinamik sistemalarining bir jinsli qo`zgalmas nuqtalari kartasi haqida so`z yuritilgan. Jahon miqyosida olib borilayotgan ko`plab ilmiy-amaliy tadqiqotlarda dinamik sistemalar keng o'rin egallaydi. Dinamik sistemalar nazariasi zamonaviy matematikaning muhim sohalaridan biridir.

**Kalit so`zlar:** matematika, modellashtirish, diskret, dinamik sistema, qo`zgalmas nuqta, tekislik, differensial tenglamalar sistemasi, metod.

Аннотация. В статье рассматривается отображение однородных неподвижных точек дискретных динамических систем. Динамические системы играют важную роль во многих научных и практических исследованиях по всему миру. Теория динамических систем - одно из важнейших направлений современной математики.

Ключевые слова: математика, моделирование, дискретная, динамическая система, неподвижная точка, плоскость, система дифференциальных уравнений, метод.

Annotation. This article deals with the map of homogeneous fixed points of discrete dynamic systems. Dynamic systems play an important role in many scientific and practical researches around the world. The theory of dynamic systems is one of the most important areas of modern mathematics.

Key words: mathematics, modeling, discrete, dynamic system, fixed point, plane, system of differential equations, method.

Introduction. Dynamic systems play an important role in many scientific and practical researches conducted worldwide. The theory of dynamic systems is explained by the fact that it is one of the most important fields of modern mathematics, firstly, it allows to reveal deep connections between different fields of mathematics, and secondly, real mechanical, physical, chemical, biological and economic processes are modeled mainly by dynamic systems.[7] The peculiarity of the theory of dynamic systems is that the mathematical model of a particular process is represented by a system of differential equations, and the study of such a system requires, in addition to the theory of differential equations, topology, differential geometry, functional analysis and even modern algebra. For this reason, the study of dynamic systems using modern methods of computational mathematics and computer technology is one of the current areas of mathematical modeling.[5]

Literature review. The concept of dynamic systems as a mathematical object was founded in the scientific work of A.Poincaré and later developed by J. Birkgoff. A.A.Andronov's scientific school also made a significant contribution to the development of the theory of dynamic systems. The scientific work of the representatives of this scientific school mainly studied the dynamic systems in the plane.[1] This direction was later followed by M.Frommer, N.N.Bautin, N.P.Erugin, L.A.Cherkas, Y.S.Ilyashenko, A.F.Andreev, a number of representatives of the Italian scientific school, S.Lefshets, F. Hartmann was also developed by the Samarkand School of Science in 1960-80 under the direction of I.S. Kukles.

Research Methodology. The research used the theory of dynamic systems, computer modeling, DN-tracking, numerical methods for solving differential equations.

Analysis and results. Let $S^{m-1}=\{x=(x_1,\ldots,x_m);x_i\geq 0;\ \sum_{i=1}^{m}x_i=1\}-$ simplex and $V:S^{m-1}\to S^{m-1}-$ quadratic stochastic operator defined by equalities

$$x_k'=x_k(1+\sum_{i=1}^{m}a_{ki}x_i),\ k=\overline{1,m}, \qquad (1)$$

where $a_{ki}=-a_{ik},\ |a_{ki}|\leq 1,$

Definition 1. $V:S^{m-1}\to S^{m-1}-$ is called a generic quadratic stochastic operator if any principal minor of even order of skew-symmetric matrix $A=(a_{ki})$ is nonzero.

It follows from Definition 1 that a quadratic stochastic operator in general position forms an open everywhere dense subset in the set of all quadratic stochastic operators.[10]

Let $X=\{x:Vx=x\}-$ the fixed points of the operator V. Obviously, $X\neq\varnothing$.

Theorem 1. If V the operator is in general position, $X-$ then there is a finite set.

Consider a complete directed graph with *m* peaks. On the edge connecting the vertices *k* and *i*, set directions from the vertex *k* to the top *i* if a $a_{ki}<0$, and the opposite direction if $a_{ki}<0$, The resulting graph is called the tournament of the dynamical system (1) and is denoted by T_m. We will adhere to the notation and definitions from [2].

Definition 2. T_m is called transitive if it has no cycles. T_m is called strong if from any top *i*, you can go to any peak *k*.

Definition 3. T_m is called homogeneous if every subtournament of it is either strong or transitive.[6]

In cases $m=2$ и $m=3$ any tournament is homogeneous. $m=4$ then there are 4 tournaments and two of them are homogeneous. When $m=5$ there are 12 tournaments, 4 of them are homogeneous. When $m=5$ there are 12 tournaments, 4 of them are homogeneous.

Definition 4. Let $\sup px = \{i : x_i \neq 0\}$ – element carrier x. Fixed point with carrier $\alpha \subset \{1, \ldots, m\}$ denote by x_α.

Take from X two fixed points: x_α и x_β. Let's say $x_2 \leftarrow - x_\beta$ if there is such γ for which $\gamma \supset \alpha \cup \beta$ and the inequality $A_\gamma x_\alpha \geq 0$ и $A_\gamma x_\alpha \leq 0$ where A is the matrix resulting from A_γ replacing all a_{ki} at $(k, i) \notin \gamma \times \gamma$. And this graph will be called the map of fixed points of system (1).

For $V : S^4 \to S^5$ let's build a map of fixed points (for homogeneous tournaments). In all cases, we count. $a_i > 0$; $i = \overline{1, 10}$.

$$\begin{aligned} x_1' &= x_1(1 + a_1x_2 + a_2x_3 + a_3x_3 + a_4x_4), \\ x_2' &= x_2(1 - a_1x_1 + a_5x_3 + a_6x_4 + a_7x_5), \\ x_3' &= x_3(1 - a_2x_1 - a_6x_2 + a_8x_3 + a_{10}x_5), \\ x_4' &= x_4(1 - a_2x_1 - a_5x_2 + a_8x_4 + a_9x_5), \\ x_5' &= x_5(1 - a_4x_1 - a_7x_2 - a_9x_3 + a_{10}x_4) \end{aligned} \quad \text{и } A = \begin{pmatrix} 0 & a_1 & a_2 & a_3 & a_4 \\ -a_1 & 0 & a_5 & a_6 & a_7 \\ -a_7 & -a_5 & 0 & a_2 & a_{10} \\ -a_3 & -a_6 & -a_3 & 0 & a_{10} \\ -a_4 & -a_7 & -a_9 & -a_{10} & 0 \end{pmatrix}$$

We put $x(a)$ – fixed point with a carrier. $\alpha \subset I$.

In this mapping, the map of fixed points has a transitive form and consists only of the vertices $x(i)$, $i = \overline{1,5}$.

Any trajectory starting inside S^4 converges to the top $x(1)$.

II. Display $V : S^4 \to S^4$ as

$$\begin{aligned} x_1' &= x_1(1 - a_1x_2 + a_2x_3 + a_3x_4 + a_4x_5), \\ x_2' &= x_2(1 + a_1x_1 - a_5x_3 + a_6x_4 - a_7x_5), \\ x_3' &= x_3(1 - a_2x_1 - a_3x_2 + a_8x_4 + a_9x_5), \\ x_4' &= x_4(1 + a_3x_1 - a_6x_2 + a_8x_3 + a_{10}x_5), \\ x_5' &= x_5(1 - a_4x_1 + a_7x_2 - a_9x_3 + a_{10}x_4) \end{aligned}$$

has 10 fixed points $x_i, (i = \overline{1,5})$ and points $\overline{x}(123)$, $\overline{x}(125)$, $\overline{x}(145)$, $\overline{x}(234)$, $\overline{x}(345)$

$$A = \begin{pmatrix} 0 - a_1 & a_2 - a_3 & & a_4 \\ a_1 & 0 - a_5 - a_6 - a_7 & & \\ -a_7 & a_3 & 0 - a_3 & a_9 \\ a_3 - a_6 & a_3 & 0 & -a_{10} \\ -a_4 & a_7 - a_9 & a_{10} & 0 \end{pmatrix}$$

Fixed point coordinates

$$M_{123}=(a_5;\quad a_2;\quad a_1;\ 0;\ 0)\frac{1}{a_1+a_2+a_5},$$
$$M_{234}=(0;\quad a_3;\ a_8;\quad a_6;\ 0)\frac{1}{a_5+a_6+a_8},$$
$$M_{125}=(a_7;\quad a_4;\ 0;\quad 0;\quad a_1)\frac{1}{a_1+a_4+a_7},$$
$$M_{345}=(0;\quad 0;\quad a_{10};\ a_9;\ a_8)\frac{1}{a_8+a_9+a_{10}},$$
$$M_{145}=(a_{10};\ 0;\quad 0;\quad a_4;\ a_3)\frac{1}{a_6+a_4+a_{10}}.$$

Solving inequalities $A_\gamma x_\alpha \geq 0,\ A_\gamma x_\alpha \leq 0,$ define pairs of fixed points

$$A\bar{x}_{123}=(0;\ 0;\ a_3a_5-a_6a_2+a_8a_1;\ -a_4a_5+a_7a_2-a_1a_9)\frac{1}{a_1+a_2+a_5},$$
$$A\bar{x}_{125}=(0;\ 0;\ a_2a_7-a_4a_5+a_1a_9;\ a_3a_7-a_6a_{42}-a_{10}a_1)\frac{1}{a_1+a_8+a_{10}},$$
$$A\bar{x}_{145}=(0;\ a_{10}a_1-a_6a_4-a_7a_3;\quad -a_2a_{10}-a_8a_4+a_3a_9;\ 0;\ 0)\frac{1}{a_4+a_8+a_{10}},$$
$$A\bar{x}_{234}=(-a_1a_8+a_6a_2-a_3a_5;\quad 0;\quad 0;\quad 0;\quad a_8a_7-a_9a_6+a_{10}a_5)\frac{1}{a_6+a_5+a_8},$$
$$A\bar{x}_{345}=(a_2a_{10}-a_3a_9+a_4a_8;\quad -a_{10}a_5+a_9a_6-a_8a_7;\ 0;\ 0;\ 0)\frac{1}{a_8+a_9+a_{10}}$$

or

$$A\bar{x}_{123}=(0;\ 0;\ 0;\ \pm y_1;\pm y_2),$$
$$A\bar{x}_{125}=(0;\ 0;\ \pm y_2;\pm y_3;\ 0),$$
$$A\bar{x}_{145}=(0;\ \pm y_3;\ \pm y_4;\ 0;\ 0),$$
$$A\bar{x}_{234}=(\pm y_1;\ 0;\ 0;\ 0;\ \pm y_5),$$
$$A\bar{x}_{345}=(\pm y_4;\ \pm y_5;\ 0;\ 0;\ 0),$$

Where

$$y_1=\frac{a_3a_5+a_6a_2+a_8a_1}{a_1+a_2+a_5},\qquad y_2=\frac{a_4a_5+a_7a_2-a_1a_9}{a_1+a_2+a_5},$$
$$y_3=\frac{a_3a_7-a_6a_4-a_6a_1}{a_1+a_4+a_7},\qquad y_4=\frac{-a_2a_{10}-a_8a_4-a_3a_9}{a_1+a_8+a_{10}},$$
$$y_5=\frac{a_8a_7-a_9a_6+a_{10}a_5}{a_6+a_5+a_8}$$

In what follows, we will use some of the concepts introduced in [2]. Depending on the signs $\{y_i\}$ we obtain the following maps of fixed points:

1) If one of $y_i>0$, the rest are less than zero, then (Fig. 1, a, b). Means, $x(234)$ hanging top. By removing it, we have a new hanging top $x(345)$. And continuing this, finally we don't get any dangling vertices.

$\varphi_{234}(x)=x_2^{y_2}\cdot x_3^{y_3}\cdot x_4^{y_4}$ - Lyapunov's leading function.

$$\varphi_{234}(x)\to 0\Rightarrow \varphi_{345}(x)\to 0\Rightarrow \varphi_{145}(x)\to 0\Rightarrow \varphi_{125}(x)\to 0\Rightarrow \varphi_{123}(x)\to 0$$

are Lyapunov functions for dynamical system (1). $\omega(x^0)\subset\{x:\varphi_{234}(x)=...=0\}$.

2) If all $y_i>0$ or all $y_i<0\left(i=\overline{1,5}\right)$, then the right side of the fixed point map will remain unchanged. The second part of the map consists of 5 vertices, oriented as

follows: $x(234)\to x(123)$,
$x(123)\to x(125)$, $x(125)\to x(145)$, $x(145)\to x(345)$, $x(345)\to x(234)$.

Here has one more fixed point $x(12345)$.

3) In other cases, the map looks like this: a part is shown in Fig. 1, and the vertices of the left side are oriented: $x(234)\to x(123)$, $x(234)\to x(345)$, $x(123)\to x(125)$, $x(345)\to x(145)$, $x(125)\to x(145)$, $x(234)-$ hanging top. By removing it, we get two dangling vertices.[6]

III. Display $V:S^4\to S^4$ as:

$$\begin{aligned}
x_1' &= x_1\left(1-a_1x_2-a_2x_3+a_3x_4+a_4x_5\right),\\
x_2' &= x_2\left(1+a_1x_1-a_5x_3-a_6x_4-a_7x_5\right),\\
x_3' &= x_3\left(1+a_2x_1+a_5x_3-a_8x_4-a_9x_5\right),\\
x_4' &= x_4\left(1-a_3x_1+a_6x_2+a_8x_3-a_{10}x_5\right),\\
x_5' &= x_5\left(1-a_4x_1+a_7x_2+a_9x_3-a_{10}x_4\right)
\end{aligned}$$

has 9 fixed points: $\bar{x}(124)$, $\bar{x}(125)$, $\bar{x}(134)$, $\bar{x}(135)$, and $x(i)\left(i=\overline{1,5}\right)$.

Definition 5. Vertices x_α и x_β are called neighboring if the following conditions are met:

1. $|\alpha\cup\beta|=|\alpha|=|\beta|$. For example, the vertices $x(124)$ и $x(125)-$ neighboring peaks.

Let's define the coordinates of these points:

$$M_{123}=(a_6;\ a_3;\ 0;\ a_1;0)\frac{1}{a_1+a_3+a_6},$$

$$M_{125}=(a_7;\ a_4;\ 0;\ 0;\ a_1)\frac{1}{a_1+a_4+a_7},$$

$$M_{134}=(a_8;\ 0;\ a_3;\ a_2;0)\frac{1}{a_2+a_3+a_8},$$

$$M_{135}=(a_9;\ 0;\ a_4;\ 0;\ a_2)\frac{1}{a_2+a_4+a_9}.$$

and

$$Ax_{124}=(0;\ 0;\ -a_2a_6-a_5a_3+a_1a_8;\ 0;\ a_6a_4-a_7a_3-a_{10}a_1)\frac{1}{a_1+a_3+a_6}$$

$$A\bar{x}_{125}=(0;\ 0;\ -a_2a_7-a_5a_4+a_9a_1;\ a_3a_7-a_6a_4+a_1a_{10};\ 0)\frac{1}{a_1+a_4+a_7}$$

$$A\bar{x}_{134}=(0;\ -a_8a_1-a_3a_5+a_2a_6;\ 0;\ 0;\ a_8a_4-a_9a_3+a_{10}a_2)\frac{1}{a_2+a_3+a_8}$$

$$A\bar{x}_{135}=(0;\ -a_1a_9+a_4a_5+a_2a_7;\ 0;\ a_3a_9-a_8a_4+a_2a_{10})\frac{1}{a_2+a_4+a_9}$$

or

$$Ax_{124} = (0;\ 0;\ \pm y_1;\ \ 0;\ \pm y_2)$$
$$Ax_{125} = (0;\ 0;\ \pm y_3;\ \pm y_2;\ 0;\ )$$
$$Ax_{134} = (0;\ \pm y_1;\ 0;\ \ 0; \pm y_4;)$$
$$Ax_{135} = (0;\ \pm y_3;\ 0;\ \pm y_4;\ 0)$$

Imagine maps of fixed points.

1) if only one of $y_i > 0$ (or $y_i < 0$) then the map of fixed points has the form a,b.

$\varphi_{135}(x) = \varphi_{125}(x) = \varphi_{124}(x) = \varphi_{134}(x) = 0$ are Lyapunov functions for dynamical system (1).

2) If a $y_1 > 0,\ y_2 > 0,\ y_3 > 0,\ y_4 > 0,$ then the right side of the penalty of fixed points G_υ is shown in Fig. 2 a.

The other part consists of four peaks: $\{x(124),\ x(125), x(134),\ x(125)\}$ co the next set of oriented edges $x(124) \to x(125), x(125) \to x(135)$, $x(124) \to x(134),\ x(134) \to x(135)\cdot$

All other cases are reduced to those considered.[9]

IV. Display $V: S^4 \to S^4$ as:

$$x_1' = x_1(1 - a_1x_2 - a_2x_3 + a_3x_4 + a_4x_5),$$
$$x_2' = x_2(1 + a_1x_1 - a_5x_3 - a_6x_4 + a_7x_5),$$
$$x_3' = x_3(1 - a_2x_1 - a_5x_5 + a_8x_4 + a_9x_5),$$
$$x_4' = x_4(1 - a_3x_1 + a_6x_2 - a_8x_3 - a_{10}x_5),$$
$$x_5' = x_5(1 - a_4x_1 - a_7x_2 - a_9x_3 + a_9x_4)$$

has 8 fixed points $x(i)\ (i = \overline{1,5})$ and points $\overline{x}(1244,\ ),\ \overline{x}(234)\ \overline{x}(245)$

$$A = \begin{pmatrix} 0 - a_1 & a_2 & a_3 & a_4 \\ a_1 & 0 & a_5 - a_6 & a_7 \\ -a_2 - a_5 & 0 & a_8 & a_9 \\ -a_3 & a_6 - a_8 & & 0 - a_{10} \\ -a_4 & -a_7 - a_9 & a_{10} & 0 \end{pmatrix}$$

$$M_{124} = (a_6;\ \ a_3;\ \ 0;\ \ \ a_1;\ 0)\frac{1}{a_6 + a_3 + a_1},$$
$$M_{234} = (0;\ \ a_8;\ \ a_6;\ \ a_5;\ 0;)\frac{1}{a_5 + a_6 + a_8},$$
$$M_{245} = (0;\ \ a_{10};\ 0;\ \ \ a_7;\ a_6)\frac{1}{a_6 + a_7 + a_{10}}$$

$$A\overline{x}_{124} = (0;\ \ 0;\ \ -a_6a_2 - a_5a_3 + a_8a_1;\ \ 0; -a_4a_6 - a_7a_3 + a_{10}a_1)\frac{1}{a_6 + a_3 + a_1},$$
$$A\overline{x}_{234} = (-a_8a_1 + a_6a_2 - a_5a_3;\ \ 0;\ \ 0;\ \ 0;\ \ -a_7a_8 - a_6a_9 + a_{10}a_5)\frac{1}{a_5 + a_6 + a_8},$$
$$A\overline{x}_{245} = (-a_1a_{10} + a_8a_7 + a_8a_4;\ 0;\ -a_5a_{10} + a_8a_7 + a_6a_9;\ \ 0;\ 0)\frac{1}{a_6 + a_7 + a_{10}}$$

or

$$Ax_{124} = (0;\ \ 0;\ \pm y_1;\ 0;\ \pm y_2),$$
$$Ax_{234} = (\pm y_1;\ 0;\ \ 0;\ \ 0;\ \pm y_3),$$
$$Ax_{245} = (\pm y_2;\ \ 0;\ \pm y_3;\ 0;\ \ 0).$$

Conclusion. Imagine maps of fixed points.

1) If all $y_i > 0$ or all $y_i < 0$, $(i = \overline{1,5})$ then the map of fixed points has the form a, b (Fig. 3)

2) If $y_1 < 0,\ y_2 > 0,\ y_3 < 0$ or $y_1 > 0,\ y_2 < 0,\ y_3 > 0$ then the right side of the fixed point map G_υ has the form. The other part consists of three peaks: $\{x(124),\ x(234),\ x(245)\}$ oriented as follows: $x(124) \to x(234),\ x(234) \to x(245),\ x(245) \to x(124)$

The uniformity of the tournament implies the uniformity of the corresponding penalty. Thus, we obtain the following statement.[3]

Theorem 2. For $\mathrm{m} = 5$ has 9 homogeneous fixed point charts for operators of Volterra type. The resulting dynamical systems are not topologically equivalent.

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